Discrete Structures. CSCI-150.

$$(A \wedge B) \equiv (B \wedge A) \quad \text{commutativity of } \wedge$$
$$(A \vee B) \equiv (B \vee A) \quad \text{commutativity of } \vee$$
$$((A \wedge B) \wedge C) \equiv (A \wedge (B \wedge C)) \quad \text{associativity of } \wedge$$
$$((A \vee B) \vee C) \equiv (A \vee (B \vee C)) \quad \text{associativity of } \vee$$
$$\neg(\neg A) \equiv A \quad \text{double-negation elimination}$$
$$(A \to B) \equiv (\neg B \to \neg A) \quad \text{contraposition}$$
$$(A \to B) \equiv (\neg A \vee B) \quad \text{implication elimination}$$
$$(A \leftrightarrow B) \equiv (A \to B) \wedge (B \to A) \quad \text{biconditional elimination}$$
$$\neg(A \wedge B) \equiv (\neg A \vee \neg B) \quad \text{De Morgan's Law}$$
$$\neg(A \vee B) \equiv (\neg A \wedge \neg B) \quad \text{De Morgan's Law}$$
$$(A \wedge (B \vee C)) \equiv (A \wedge B) \vee (A \wedge C) \quad \text{distributivity of } \wedge \text{ over } \vee$$
$$(A \vee (B \wedge C)) \equiv (A \vee B) \wedge (A \vee C) \quad \text{distributivity of } \vee \text{ over } \wedge$$
$$A \wedge \text{ True} \equiv A \quad \text{identity}$$
$$A \vee \text{ False} \equiv A \quad \text{identity}$$
$$A \vee \text{ True} \equiv \text{ True} \quad \text{domination}$$
$$A \wedge \text{ False} \equiv \text{ False} \quad \text{domination}$$
$$A \vee \neg A \equiv \text{ True} \quad \text{complementation (excluded middle)}$$
$$A \wedge \neg A \equiv \text{ False} \quad \text{complementation (non-contradiction)}$$
$$A \wedge A \equiv A$$
$$A \vee A \equiv A$$

$$\frac{A}{A \vee B} \quad \text{“}\vee\text{-Introduction”}$$

$$\frac{\neg B \qquad A \to B}{\neg A} \quad \text{“Modus Tollens”}$$

$$\frac{A \qquad B}{A \wedge B} \quad \text{“}\wedge\text{-Introduction”}$$

$$\frac{A \to B \qquad B \to C}{A \to C} \quad \text{“Hypothetical Syllogism”}$$

$$\frac{A \wedge B}{A}, \quad \frac{A \wedge B}{B} \quad \text{“}\wedge\text{-Elimination”}$$

$$\frac{A \vee B \qquad \neg A}{B}, \quad \frac{A \vee B \qquad \neg B}{A} \quad \text{“Disjunctive Syllogism”}$$

$$\frac{A \qquad A \to B}{B} \quad \text{“Modus Ponens”}$$

$$\frac{A \vee B \qquad \neg A \vee C}{B \vee C}, \ \ldots \quad \text{“Resolution”}$$

$$\frac{\text{assuming } A\text{, we infer } B}{A \to B} \quad \text{“}\to\text{-Introduction” (Deduction theorem)}$$

$$\frac{\text{assuming } A\text{, we infer a contradiction}}{\neg A} \quad \text{“Proof by contradiction”}$$