Discrete Structures. CSCI-150. Spring 2017.

Homework 7. Due Mon. Mar. 27, 2017.

A note:

We can use the notation (x **rem** y) to denote the **REMAINDER** when x is divided by y.

For example:

$$1652 \textbf{ rem } 100 = 52,$$
$$\text{because } 1652 = 16 \cdot 100 + \underbrace{52.}_{\text{the remainder}}$$

(Lehman and Leighton's book is using **rem** to denote this operation, while Rosen is using **mod**. I encourage you to use the **rem** symbol for the remainder in this class, and the **mod** symbol will be used for a slightly different purpose. You will see on Monday.)

Problem 1

For $a, b \in \mathbb{Z}$, prove that if $a \mid b$ and $b \mid a$ then $a = b$ or $a = -b$.

Problem 2

For positive $a, b, c \in \mathbb{Z}$, prove that if $c = \gcd(a, b)$ then $c^2 \mid ab$.

Problem 3 (Graded)

Prove that if positive integers a and b are odd then $2 \mid (a^2 + b^2)$, but $4 \nmid (a^2 + b^2)$.

Problem 4 (Graded)

Prove that for all positive integers n:

$$3 \mid (n^3 + 2n).$$

It can be done either by induction, or by cases.

The proof by induction is standard. If you decide to prove it by cases, consider the remainder (n **rem** 3), it can be equal to 0, 1, or 2, so we can say that for any n: $n = 3k$, or $n = 3k + 1$, or $n = 3k + 2$.

Problem 5 (Graded)

Using Euclidean algorithm, compute

(a) $\gcd(244, 28)$ (b) $\gcd(323, 177)$

Write each step of the algorithm execution.