

Discrete Structures. CSCI-150. Spring 2017.

Homework 4.

Due Mon. Feb. 27, 2017.

Problem 1 (Graded)

A computer science professor has eleven different programming books on their bookshelf. Five of them deal with the programming language C++, and the other six deal with LISP. In how many ways can the professor arrange these books on the shelf

- (a) if there are no restrictions?
- (b) if all the C++ books must be next to each other?
- (c) if all the C++ books must be next to each other and all the LISP books must be next to each other?
- (d) if the languages should alternate?

Note that all the books are distinct. Not for grade, you may consider the case when two of the C++ books are identical copies. Then, what if all the C++ books and all the LISP books are identical copies?

Problem 2

How many permutations of the letters ABCDEFGH contain

- (a) the string AB
- (b) the strings AB and FGH
- (c) the string AB or the string FGH

Think carefully when solving the last question. A hint: $|A \cup B|$.

Problem 3 (Graded)

- (a) Count the total number of bitstrings of length 19.
- (b) Count the number of bitstrings composed of seven 0s and twelve 1s.
- (c) Count the number of bitstrings composed of seven 0s and twelve 1s, such that every 0 must be immediately followed by a 1.
(An example of a good bitstring: 1101110110101010101.)
- (d) Amy wrote 19 short stories: 12 happy and 7 sad. To publish a book, she has to choose in what order the stories should be put in it. She decided that each sad story should be immediately followed by a happy one. In how many ways can Amy order her stories?
(Hint: The answers for (c) and (d) are not the same.)

Problem 4 (Graded)

A pizzeria offers 777 types of pizza and 3 types of soda. Mary goes there everyday for lunch, always buying one slice of pizza and one soda. However, she never gets exactly the same thing on two consecutive days (that is, each time, either the drink or the pizza (or both) is different from what she had yesterday).

In how many ways can she plan her lunch for the next 15 days if today she tried a different pizzeria and did not like that place at all?

Answer: approximately 3.240×10^{50} (but you should try to find the exact formula, not an approximation).

Problem 5 (Graded)

How many different sets can be made out of 5 possible elements: a, b, c, d, e ? Don't forget to count the empty set (that contains none of these elements).

Problem 6

Prove that

$$\binom{5}{0} + \binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5} = 2^5.$$

Problem 7

Ellen draws 5 cards from a standard deck of 52 cards.

- (a) In how many ways can her selection result in a hand with no hearts?
- (b) A hand with at least one card of hearts?

Problem 8 (Graded)

Prove that

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}.$$