Discrete Structures. CSCI-150. Spring 2017.

Homework 3. Due Wed. Feb 22, 2017.

Introduction

Always fully explain your solutions. Answers by themselves are useless and don't prove anything. In this homework, try to refer to the rule of summation, and the rule of product, when you are using them.

When solving a combinatorial problem, for example counting the number of certain objects (bitstrings, groups of people, etc.), always **try to think how you generate one instance** of such an object. Analyze this generation process; ask yourself: **When exactly do I make a choice?**

An example. Count the number of license plates of the following format: 1 or 2 letters, followed by 1, 2, or 3 digits.

There are many ways to generate such a license plate. Consider the following two methods:

(a) Method 1. We can first, generate the letters. Then generate the digits.

We have to do both subtasks. So, once we know in how many ways we can do each of the subtasks, we can, by the rule of product, multiply the numbers and obtain the answer.

Subtask 1. To generate the letters, there must be either 1 or 2 letters. By the rule of sum,

$$L = 26 + 26^2 = 702.$$

(here, 26 is the number of ways to pick 1 letter, and $26 \cdot 26 = 26^2$ is the number of ways to pick a pair of letters)

Subtask 2. To generate the digits, there can be 1, 2, or 3 digits:

$$D = 10 + 10^2 + 10^3 = 1110.$$

Therefore, the number of ways to generate a license plate is

$$L \cdot D = 702 \cdot 1110 = 779220.$$

(b) Method 2. There are 6 possibilities for a license plate:

1 letter + 1 digit	$26 \cdot 10 = 260$
1 letter + 2 digit	$26 \cdot 10^2 = 2600$
1 letter + 3 digit	$26 \cdot 10^3 = 26000$
2 letter + 1 digit	$26^2 \cdot 10 = 6760$
2 letter + 2 digit	$26^2 \cdot 10^2 = 67600$
2 letter + 3 digit	$26^2 \cdot 10^3 = 676000$

Because all these 6 cases correspond to the disjoint sets of license plates (Do you agree? What does that mean that they are disjoint?), we add the numbers up by the rule of sum, and get

$$260 + 2600 + 26000 + 6760 + 67600 + 676000 = 779220.$$

As expected, both methods give the same answer.

Your solution for each problem should have a complete explanation of your reasoning. It does not have to be a super long text, but it should explain your reasoning.

A rule of thumb: If you give your solution to your younger brother or sister, they should be able to understand your explanation.

Problem 1

(a) Count the number of bitstrings of length 11.

(b) In how many ways you can paint 11 rooms, if you have two types of paint: white and beige? (Mixing the paint is not allowed).

(c) In how many ways you can paint the same 11 rooms with 17 types of paint.

(d) What if there are R rooms and N types of paint?

Problem 2 (Graded)

Assume that at a certain university, there is the department of Mathematics (with 10 faculty members), the department of Computer Science (15 faculty members), and the department of Economics (20 faculty members).

In how many ways can we select a committee if there should be

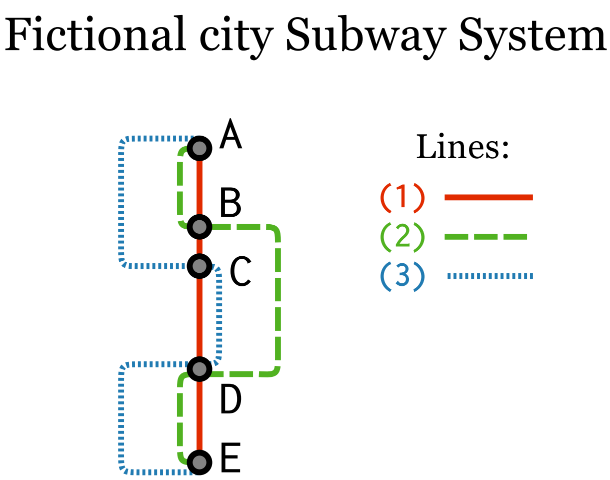
(a) one person from the Math department, and one from either CS or Economics.

(b) exactly 1 representative from each department.

(c) 2 persons, and they should be from different departments.

(d) 1, 2, or 3 persons but no two from the same department.

Problem 3 (Graded)



In the figure above, you can see the map of a fictional subway system. There are 3 train services: (1), (2), and (3). All transfer stations are labeled with the uppercase letters.

The stations A and E are the terminals for all three trains.

If we have to count the number of ways to travel from A to E without transfers then there are, obviously, 3 ways to do so.

(a) Count the number of ways to travel from the station A to the station E, when you transfer from one train to another exactly once.

(b) The segment CD of the line (1) was closed due to construction. Repeat the task again, count the number of ways to travel from A to E with exactly one transfer.

(In case you have spare time, you may try to count the number of ways to travel from A to E transferring exactly twice).

Problem 4 (Graded)

Let us call the following six letters $\{A, E, I, O, U, Y\}$ **vowels**, and the other twenty letters be **consonants**.

Also, we say that a **name** is an alternating sequence of vowels and consonants (so possible name templates are: v, c, vc, cv, vcv, cvc, vcvc, cvcv, and so on).

Please **count the number of names** of

(a) length 3.

(b) length 3 that are palindromes (Such as Ada or Bob).
(A *palindrome* is a string whose reversal is identical to the string.)

(c) length 4 that are palindromes.

(d) length 4 that are *not* palindromes.

(e) length 7 that are palindromes.

(f) length 5, where each letter can be used at most once. (That is, Emina and Petar are good, while Hasan and Elise are bad.)

Problem 5 (Graded)

First, count the total number of bitstrings of length 10.
Then, find how many bit strings of length 10 either begin with four '0's or end with two '1's.

Problem 6

How many five-digit integers (in the conventional base-10 numeral system)

(a) start with a '9'?

(b) contain a '9'?

(c) do not contain a '9'?

Note that a five-digit number is different from a string of five digits. Do you see in what way?

Problem 7 (Graded)

There are 10 professors at a certain CS department. According the tentative course schedule, there are 7 distinct courses that should be taught next semester.

Please count in how many ways the teaching assignments can be distributed among the department faculty if:

(a) each professor should teach at most one class?

(b) all 7 classes must be taught by either Professor Lamport or Professor Papadimitriou.

(Clarification: In both cases, you cannot assign two professors to teach the same class together.)