Discrete Structures. CSCI-150. Spring 2017.

Homework 12. **Due Wed. May 17, 2017.**

Trees

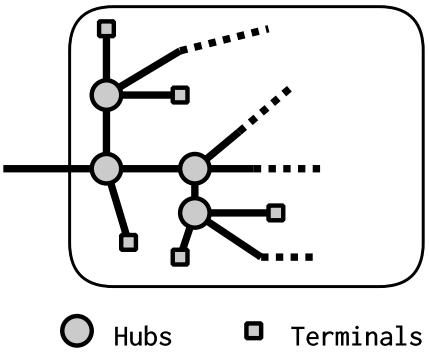
Problem 1

Suppose 1000 people enter a chess tournament. Use a rooted tree model of the tournament to determine how many games must be played to determine a champion, if a player is eliminated after one loss and games are played until only one entrant has not lost. (Assume there are no ties.)

Problem 2 (Graded)

Amtrak plans to extend their railroad network to a big island, which is connected to the continent by a bridge.

According to the plans, there will be M stations on the island. There are two types of stations: the first type are hubs that connect 4 railroads, the second type are dead-end (terminal) stations, with only one railway line. To reduce the costs, the railroads don't make loops, that is, there are *no simple cycles* in the network, so the system is cheaper, although all stations are connected. Only one of the hubs is directly connected to the outside world.



How many hubs, and how many terminals will be built? (The total number of stations is M).

Problem 3 (Graded)

Use Huffman coding to encode these symbols with given frequencies:
A: 0.05, B: 0.07, C: 0.08, D: 0.10, E: 0.15, F: 0.25, G: 0.30.

Show all intermediate steps. What is the average number of bits required to encode a symbol?

Probability

Problem 4 (Graded)

Three cards are drawn from a standard 52-card deck. Each combination of three cards was equally likely.

Find the probability that the drawn hand is

(a) $\{K\spadesuit, Q\heartsuit, J\diamondsuit\}$ (a hand is a set, that is, the card order does not matter).

(b) King, Queen, and Jack of any suit.

(c) At least one Ace.

Problem 5 (Graded)

You have 10 books on your bookshelf. They are arranged in the order of increasing number of pages (from the thinnest to the thickest):

5 pages, 10 pages, 20 pages, 40 pages, 80 pages, ... 2560 pages

(so, every subsequent book is twice as thick as the previous).

Your grandmother left an important note on one of the pages of those books, but you don't know the book and the page.

(a) Assuming she could choose any page with equal probability, what is the probability that the note is in the book #4?

(b) What's the probability that the note is in one of the thinner books (#1 – #5)?

(c) In one the the thicker books (#6 – #10)?

Problem 6

A project was implemented by three developers: Alice, Bob, and Carol. They used four languages: C, C++, Python, and JavaScript. The table summarizes what fraction of the code was written by each person in each language.

	C	C++	Python	JavaScript
Alice	5/24	1/8	1/6	0
Bob	1/24	1/8	1/12	0
Carol	0	0	1/12	1/6

You pick a piece of code at random.

(a) Who is most likely to be the author of that piece of code?

(b) Who is most likely to be the author given that it was written in JS?

(c) Who is most likely to be the author given that it was written in C or C++?

(d) What is the probability that it was written by Bob? Does the probability change if we know that the code is in Python? Are the events $Python$ and Bob independent or not?

(e) Are the events $Alice$ and C independent?

(f) The same question for $Carol$ and JS.

Problem 7 (Graded)

You flip a **biased coin** n times. The probability of getting "heads" in one flip is p, and the probability of "tails" is $1 - p$.

Please find

(a) the probability that there were no heads in n flips,

(b) the probability of getting heads exactly once in n flips,

(c) the probability of *at most one* heads,

(d) the probability of *at least two* heads.

(These n successive coin flips are identical to n independent Bernoulli trials with the probability of a success in one trial equal to p.)