Discrete Structures. CSCI-150. Spring 2017.

Homework 11. Due Mon. May 8, 2017.

Problem 1

For each course at a university, there may be one or more other courses that are its prerequisites. How can a graph be used to model these courses and which courses are prerequisites for which courses? Should edges be directed or undirected? Looking at the graph model, how can we find courses that do not have any prerequisites and how can we find courses that are not the prerequisite for any other courses?

Problem 2 (Graded)

Draw these graphs: (a) K_7, (b) $K_{2,5}$, (c) C_7, (d) Q_4.
All of these special graphs are described in Rosen, K_n is the complete graph, $K_{n,m}$ is the complete bipartite graph, C_n is the cycle graph, and Q_n is the hypercube graph.

How many vertices is in K_n, $K_{n,m}$, C_n, Q_n?

Problem 3 (Graded)

A simple graph is called regular if every vertex of this graph has the same degree. A regular graph is called n-regular if every vertex in this graph has degree n.

Recall thatK_n is the complete graphs with n vertices. And $K_{m,n}$ is the complete bipartite graph (see the definition in the book).

(a) Is K_n regular?

(b) For which values of m and n graph $K_{m,n}$ is regular?

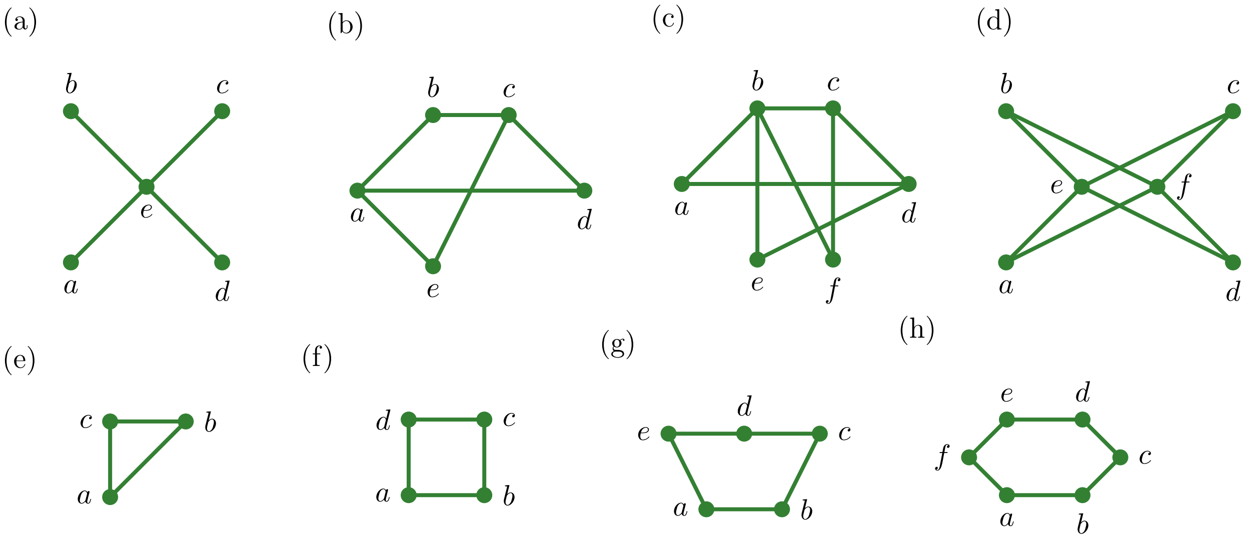
(c) How many vertices does a 4-regular graph with 10 edges have?

Problem 4 (Graded)

Assuming the friendship relation is always mutual (i.e. symmetric):

Prove that in any group of $n \geq 2$ people, there are at least 2 persons with the same number of friends in the group.

Problem 5



We know that a graph is bipartite if and only if it is 2-colorable.

For the graphs given above, either prove that they are bipartite by showing that they are 2-colorable, or prove that they are not 2-colorable, and so they are not bipartite.

Problem 6 (Graded)

A simple graph is called n-regular if every vertex of the graph has degree n.

Show that in an n-regular bipartite graph $G = (V, E)$ with a bipartition of the vertex set (V_1, V_2), the sets in the bipartition V_1 and V_2 contain the same number of vertices, $|V_1| = |V_2|$.

Problem 7 (Graded)

Given a simple graph with n vertices, prove that <u>if</u> the degree of each vertex is at least $(n-1)/2$ <u>then</u> the graph is connected.

Hint: First, you may consider a small graph, for example a graph with 5 vertices, can you make it disconnected? Then generalize to the general case with n vertices.

Problem 8

For which values of n, does the complete graph K_n have an Euler cycle?
For which values of n and m, does the complete bipartite graph $K_{n,m}$ have an Euler cycle?

Problem 9

What are the adjacency matrix and the adjacency list of a graph?
Find the adjacency matrix of the graph shown in the figure.
Find the adjacency list of the graph.